Quantitative Analysis

This section introduces students to some basic quantitative skills for the sciences, including measurement, dimensional analysis, scientific notation, unit conversions, reporting of answers, and general problem solving skills.

Measurement and Quantitative Skills

Much of science involves measurement. In this class, you will be asked to do some measuring or to manipulate numbers obtained by others. When you make measurements, it is important to report the unit of measure used.

A **unit** is the label put on a number to show what that number actually represents. For instance, if you measure the length of a wall using a tape measure and you report that the wall is three long, this doesn't tell you much. One would ask, three what? Three feet long? Three yards long? Three inches long? Three meters long? Three inches is very different than three yards. Feet, yards, inches, meters are all **units of measure**. From this example you can see that it is vital to report the units of measure so that one can understand what the number means in actuality.

Systems of Measurement:

There are two major systems of measurement used in the United States: the Customary system and the Metric system. The Customary system is probably the one that you have seen the most. It uses inches and feet, ounces and pounds, quarts and gallons, etc. The rest of the world uses the Metric system. Because scientific reports are often shared around the world, American scientists must learn to use the Metric system as well.

Americans like the Customary system because it is the one they know. The Metric system, however, is an easier system to use. The Metric system has only one basic unit for each dimension. For example, the unit of mass is the gram, and the unit of length is the meter. Time is the only exception. It is measured the same in both systems, so seconds, minutes, hours, days, etc. are the units of time in the Metric system, just as they are in the Customary system.

In the Customary system, you choose the unit you will use by the amount to be measured. Short lengths are measured in inches, medium lengths in feet or yards, and long lengths in miles. To convert from one unit to another you have to remember many numbers. Inches to feet is a factor of 12. Feet to yards is a factor of 3. Feet to miles is a factor of 5,280. More information about how to convert units is given later.

In the Metric system, all that changes is the power of ten. Small lengths are measured in millimeters (1/1000th or 10^{-3} meters) or centimeters (1/100th or 10^{-2} meters). Medium lengths are measured in meters. Longer lengths are measured in kilometers (1000 or 10^{3} meters).

In the Metric system prefixes are given to the powers of ten. Here is a list of some powers and their prefixes:

Power	Prefix
10^{-9}	nano
10^{-6}	micro
10^{-3}	milli
10^{-2}	centi
10^{-1}	deci
10^{0}	no prefix. This is the basic unit.
10^{1}	deca
10^{2}	hecta
10^{3}	kilo
10^{6}	mega

Once you understand scientific notation, the Metric system is also easy to understand and to use. The two most common problems Americans have with the Metric system are converting Customary units to Metric units, and trying to understand the amounts signified by the Metric units. We know the approximate length of an inch and a foot. We do not intuitively know how long one meter is. This intuition will improve as you spend more time working with the Metric system.

Dimensional Notation:

All things can be measured in terms of length, mass, time, and temperature. We will not focus at this time on temperature, but concentrate on the other three dimensions of measurement. Length, mass, and time are often symbolized by L, M, and T, respectively. All quantities that we measure consist of some combination of these three dimensions. Measurements may be made in either the Customary System or the Metric system (see section below). The numbers will differ, but what you actually measure is the same. The conversion table given to you contains the different units for measuring each of the dimensions.

The Length Dimension **(L)** deals with anything measured as a length, area, or volume. Length is the specified distance between two points, or the distance around a circle. Examples of length include feet, meters, yards, kilometers, and miles. The surface area of a lake, or the area of a piece of land requires knowing more information. Area is length times width and is denoted by L^2. Examples of area include ft^2, cm^2, acre, and hectare. Volume is the space occupied by something in three dimensions, or length times width times height. Volume is denoted by L^3. Examples of volume include ft^3, m^3, and acre-ft. It is very important to remember that liters and gallons are measures of volume, **not** weight or mass and are therefore denoted by L^3.

The Mass Dimension (M) involves a straight-forward but sometimes misunderstood concept. We often think of mass and weight as being the same thing. They are not. Mass is a property of matter. It refers to the amount of matter contained in an object. The mass of an object will be the same anywhere in the universe. Weight is a measure of the gravitational pull on an object. While this is related to the mass of the object, it also depends on gravity. Because gravity differs around the universe, the weight of objects differs as well. For instance, your mass is the same whether

you stand at sea level, on a mountain top, or on the moon. Your weight, however, would be greatest at sea level, slightly less on a mountain top, and much less on the moon, since gravity changes. Because there is so little change in weight across the surface of the earth, we usually measure weight and use that information in place of mass. Keep in mind, however, that it is actually mass that we are interested in. Examples of mass include lb, kg, and ton.

The <u>Time Dimension</u> **(T)** is the same in both the Customary and the Metric Systems. Seconds are seconds, and hours are hours, etc.

Examples of Typical Quantities Represented in Dimensional Notation

Quantity	**L,M,T Notation**	**Examples**
Length	L	inches, feet, meters, miles
Area	L^2	ft^2, m^2, mi^2, acre, hectare
Volume	L^3	ft^3, cm^3, m^3, gal, ml, acre-ft
Velocity	L/T	mi/hr, ft/s, in/yr
Acceleration	L/T^2	m/s^2, ft/s^2
Force	ML/T^2	$kg*m/s^2$
Concentration	M/L^3	mg/L, g/L, g/ml
Discharge or Flow Rate	L^3/T	ft^3/s, m^3/s, gal/day
Rainfall Intensity	L/T	mm/s, in/hr
Gradient or Slope	L/L = 1 (dimensionless)	ft/ft, ft/mi

<u>How to Put Units of Measure Into Dimensional Notation</u>

Dimensional notation is used when you are interested in WHAT is being measured, but you do not need to know specific numbers or units. **To convert to dimensional notation, write the appropriate dimensions (combinations of L, L^2, L^3, M, and T) in place of the given units.** For example, let's say we want to turn the <u>units</u> of

$$\frac{m^3 \text{ x lb x sec x ft}}{\ell \text{ x acre-ft x } in^2}$$

into dimensional notation. The first step would be to examine each unit and determine what dimension it represents. At first you may want to use the conversion table to find each unit and its dimension, but eventually it will become easier to do in your head. In this case, m^3 is a meter cubed. This is a volume and is represented by L^3. lb is a pound and is a unit of mass represented by M. Sec represents a second which is a unit of time (T). Ft is a length (L). ℓ is a liter and is a unit of volume (L^3). Acre-ft is also a volume (L^3). Lastly, in^2 is inches squared, a unit of area (L^2). After substitutions are made, the dimensions look like this:

$$\frac{L^3 \text{ x M x T x L}}{L^3 \text{ x } L^3 \text{ x } L^2}$$

and can be simplified. **Remember that when you multiply numbers with powers, the powers are added. When you divide numbers with powers, the powers are subtracted.** For example, $L^3 \times L^2 = L^5$. $L^5 / L^2 = L^3$. Simplifying the multiplication, we get:

$$\frac{L^4 \times M \times T}{L^8}$$

Dividing, we get:

$$\frac{M \times T}{L^4}$$

One common mistake is to write L (length) for ***l*** (liter). A liter is actually a volume, and should by symbolized by L^3. Another common problem is the distinction between units of volume and units of mass. Things that have volume also have mass, so they can be confusing. Remember that volume is the measurement of how much space an object occupies, and mass is the measurement of the amount of matter within the object. A gallon, for example, is a unit of volume. A gallon of gas has a different mass than a gallon of water or a gallon of oil, so a gallon cannot be a unit of mass. But a gallon of anything takes up the same amount of space so gallon is volume, L^3.

One use of dimensional notation is to check for errors in calculations. If your answer is not in the correct dimensions, you need to go back through your work and look for your error. Let's look at the following example problem: You need to drive 300 miles, and you are driving 60 miles/hour. How much time will it take you to reach your destination? This questions asks 'how much time', so you know the answer must be in units of time. Suppose you accidentally worked the problem like this:

300 miles * 60 miles/hr = 18000 miles^2 / hr.

Miles^2 / hr is the dimension of L^2/T, and you needed an answer in just T. You know there has been a mistake. Or suppose you worked the numbers correctly, but made a mistake with the units:

300 miles / 60 miles/hr = 5 / hr.

Again, you would know there was an error. These are simple examples, and these mistakes would be obvious enough to correct as you went along. However, it is a good idea when working more complex problems to check the dimensions of your answer to the dimensions suggested in the question.

Scientific Notation:

Scientific Notation is a method of writing numbers as powers of ten. Very small and very large numbers are more easily written in scientific notation. For example, it is difficult to keep track of all the zeros in 0.000000003574. This is more easily written and understood as 3.574×10^{-9}. 75,243,000,000,000 is better written as 7.5243×10^{13}.

There are some specific rules about writing numbers in scientific notation. Let's use the example of putting 15,866 into scientific notation. When no decimal is written, it is implied at the end of the number **(15,866.)**. Move the original decimal point so that there is only one number to the left of the decimal place and for the time being, rewrite all other numbers **(1.5866)**. To determine the power of ten, go back to the original number and count the number of places that you had to move to get to that decimal place. **In this case, we moved the decimal four places**. If you move the decimal to the left (if the original number is greater than 1), the power will be positive. If you move the decimal to the right (if the number is between 0 and 1), the power will be negative. **Now we have 1.5866×10^4**. Lastly, round the number to two decimal places. **Your final answer would be reported as 1.59×10^4**. If the number contains only zeroes after the decimal point, include two of them as your two decimal places. For example, 5,000,000 becomes 5.00×10^6.

Here are several examples of numbers as you are used to seeing them, and how they appear in scientific notation:

Number	Scientific Notation
0.0003682	3.682×10^{-4}
0.003682	3.682×10^{-3}
0.03682	3.682×10^{-2}
0.3682	3.682×10^{-1}
3.682	3.682×10^{0}
36.82	3.682×10^{1}
368.2	3.682×10^{2}
3,682	3.682×10^{3}
36,820	3.682×10^{4}
36,820,000,000,000	3.682×10^{13}

Using Your Calculator: Scientific Notation and Exponents

Some calculators function differently than others when it comes to exponents and scientific notation. Be careful when entering exponents. Use the y^x button, not the ln button (natural log). For example, when entering 4^8, first enter 4, then hit the y^x button, then enter 8. Other calculators use the carrot (^) for exponents. For example, you would enter 4^8.

Regarding scientific notation, many calculators report answers using E to denote "times 10 to the power". For example, if the calculator reports 8E12 this means 8×10^{12}. If you need help using your calculator, ask your TA.

Unit Conversions:

Hydrology is a science that uses a tremendous number of measurements. It is very data intensive. The data come from many sources, and are collected under many different circumstances. Sometimes it is convenient to measure time in seconds, sometimes in minutes, sometimes in days, weeks, months, or even years. The units often depend on the instrument that is used to take the measurement. Small scales may measure mass in grams, large scales may be marked in pounds or even tons. Often we need to compare results having different units. Therefore it is essential to master the technique of converting from one unit to another.

On a more personal note, unit conversions are very helpful if you travel to a foreign country. The currency exchange rate is probably the conversion you would be most concerned with. Other

conversions that might be helpful are the speed limit, the temperature, and the weights or volumes of items you buy.

Units are much like numbers. They can be multiplied, squared, cubed, divided, and the square root taken. You can multiply and divide numbers that have different units, but **you cannot add and subtract them unless the units are the same.** For example, 300 miles divided by 5 hours gives you 60 miles/hr, but 300 miles minus 5 hours doesn't make sense.

The single most important thing to remember when converting units is to **always write down the units** that go with the numbers. Many problems require you to convert hours to seconds, feet to meters, etc., and the most common mistake that students make is losing track of the units. **A number is meaningless unless it is described by some unit.** If you are told to go 14 to get to someone's house, what does that mean? 14 blocks is very different from 14 miles. The units give the number some reference. Unit conversions will seem very simple to some of you, and it will be tempting to hurry through problems without writing down the units. This is not recommended, since losing track of the units makes it very easy to make silly mistakes like the one above.

Converting Units

When converting units, different people may be comfortable with slightly different formats or work the problem in a slightly different order, but the total work done and the answer obtained will be the same. **No matter how you do the problem, you must show your work.** Perhaps the easiest way to do unit conversions is to multiply the original number by a series of unit conversion factors that can be found on your unit conversion sheet. This is the method we recommend and will demonstrate below.

EXAMPLE: Convert the following.

$33\,ft/hr \quad = \qquad ? \qquad cm/\text{sec}$

STEP 1: Write down what you know. If the units are shown on a single line, re-write them so that you can clearly keep track of what is on the top and what is on the bottom.

$$33\,\frac{ft}{hr}$$

STEP 2: Keep in mind your ultimate goal.

You want to multiply the original number by a series of conversion factors.

$$\frac{33\text{ ft}}{1\text{ hr}} \quad \text{x} \quad \frac{??}{??} \quad \text{x} \quad \frac{??}{??} \quad = \quad \underline{\hspace{3cm}} \quad \frac{\text{cm}}{\text{sec}}$$

You must fill in the conversion factors so that you get rid of the original units but get the new units back out. Since this is multiplication, use the form of the conversion factor that will allow you to cancel out each of the old units and give you the new ones.

STEP 3: Figure out which units you are converting to which units and find the conversion factors to accomplish the above.

This is where dimensional notation can help. We want to change the Length Dimension of **feet into centimeter**. We also need to change the Time Dimension from **hour to second**.

The next step is to find the conversion factors that relate each set of units. Use your unit conversion sheet to look up the conversion factors. Do one step at a time. First, let's find the conversion for hour to seconds.

Hour to seconds: 1 hour = 3600 seconds

Next, let's find the conversion factor for **feet to centimeters**. We find that the conversion factor appears twice. We have :

Feet to centimeters: 1 ft = 30.48 cm and also 1 cm = 0.0328 ft.

Do not let this confuse you. These are really saying the same thing, and either one used properly will give you the right answer. 1/30.48 = 0.0328, so one conversion factor is simply the inverse of the other one. For this problem, let's use the conversion factor of 1 ft = 30.48 cm.

STEP 4: Fill in the appropriate form of the conversion factors.

We will be multiplying the original number (33 ft/hr) by two conversion factors. How do we decide which way the fraction must be written? We need to look at the original units and units needed at the end.

First, let's figure out the **form** of the conversion factor to use to go from feet to centimeters.

1 ft = 30.48 cm can be written fractionally in two different ways:

$$\frac{1\,ft}{30.48\,cm} \quad or \quad \frac{30.48\,cm}{1\,ft}$$

Likewise, 1 hour = 3600 seconds can be written fractionally in two different ways:

$$\frac{1\,hr}{3600\,\mathrm{sec}} \quad or \quad \frac{3600\,\mathrm{sec}}{1\,hr}$$

Since feet is in the numerator (top) of the original number, we want to use the form of the conversion factor that has feet in the denominator (bottom) so **feet** cancel out giving us centimeters. Likewise, since hour is in the bottom in the original number, we want to use the form of the conversion factor that has hour in the top so **hour** cancel out, giving us seconds. It does not matter which conversion is done first. To avoid skipping a conversion, it is usually easiest to go in the same order as the original units, working left to right and top to bottom.

$$33\ \frac{ft}{hr} \quad x \quad \frac{30.48\,cm}{1\ ft} \quad x \quad \frac{1\ hr}{3600\ \text{sec}} = \quad ?$$

STEP 5: Perform the appropriate mathematical calculations and retain the appropriate units.

Multiply and divide the numbers. Cancel the units that can be cancelled and retain the remaining units. Circle your answer.

$$33\ \frac{ft}{hr} \quad x \quad \frac{30.48\,cm}{1\ ft} \quad x \quad \frac{1\ hr}{3600\ \text{sec}} = \quad 0.2794\ \frac{cm}{\text{sec}}$$

Reporting of Answers

Rounding

Calculators show anywhere from 8 to 12 numbers on their display. Therefore, as you work problems, your numbers will contain 8 to 12 digits. Depending on the size of the number, this may include numbers after the decimal point. For example, if you are converting 53 inches into feet, your calculator may show that 53/12 = 4.41666666666 feet. This is correct. If you measured exactly 53 inches (53.000000 inches, for example), then the same length measured in feet would be exactly 4.41666666666. However, the measurement of 53 inches was probably approximate. It was probably rounded to the closest inch or half inch. If you report 4.4166666666 ft as the length, it may mislead people into thinking that you were actually able to measure the length that precisely. In reality, very few instruments in the world can measure that precisely.

Therefore, scientists keep all these digits during the calculation, and then round off their final answers to reflect this uncertainty. There is a complicated way of deciding exactly how to round off. Scientists call this the method of significant figures, and it involves the precision of the instruments used to do the measurements. In this class we will use simpler yet reasonable rules for reporting of answers:

1. **Keep all digits during all stages of the calculation. Only round off the final answer.**

2. **As a general rule, when rounding off, keep the first 3 significant digits, regardless of the size of measurement.** "Real" digits do not include zeros that are used only as placeholders. Look at the fourth significant digit to decide whether to round up or down. Round up if the fourth digit is 5 or more. Let's go through some examples:

243,867.25 — The first four significant digits are 2438. We keep the first three digits, which are 243, and look at the fourth digit to decide if the 3 should be rounded up. The fourth significant digit is an 8. Therefore, we round the 3 up to a 4. Our final answer to report is 244,000 although you may leave the numbers 243,867. In either case, do not include the digits to the right of the decimal because this number is so big, that amount is negligible in the grand scheme of things.

0.00350923	The first three significant digits are 350. The zeros before the three are only placeholders. The zero between the 5 and the 9 has real value, and counts as a significant digit. The fourth digit is 9, so we round up. Our final answer would be 0.00351. We must still use those placeholders, but they do not count as any of the three significant digits.
25.644	The first three significant digits are 256. After the 6 is a 4, so we do not round up. 25.6 will be the reported answer.

This is easiest to see in scientific notation. All answers will be #.## x $10^{\#}$. Our three examples would be written as 2.44 x 10^5 , 3.51 x 10^{-3}, and 2.56 x 10^1. Below are some more examples. Please read through them carefully and make sure you understand why they are written as they are.

Number as calculated	Number as written for final answer	Number rewritten in scientific notation
45.29380	45.3	4.53 x 10^1
2,293,472	2,290,000	2.29 x 10^6
0.002293472	0.00229	2.29 x 10^{-3}
335.6826	336	3.36 x 10^2
472,036	472,000	4.72 x 10^5
471,993	472,000	4.72 x 10^5
472,543	473,000	4.73 x 10^5
472,499	472,000	4.72 x 10^5
0.3306	0.331	3.31 x 10^{-1}
533,245,679,326	533,000,000,000	5.33 x 10^{11}
0.00000005332457	0.0000000533	5.33 x 10^{-8}
3.2588	3.26	3.26 x 10^0

3. Always write down the units throughout your work, including the final answer.

Points may also be deducted for answers without units!!!

In summary, when doing problems and calculations in this class, follow these rules when you report results:

1. Round answers to three significant digits.
2. If asked to report your answers in scientific notation, round answer to three significant digits and then report final answer to two decimal places.
3. Always include the **units** next to the number.
4. Circle your final answer.